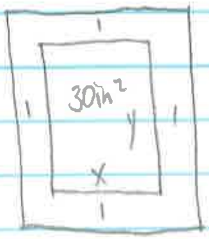
Optimization Solutions

p. 220   17, 19, 22   and the Rancher problem

17.

$\boxed{2}$



looking for minimum amount of paper

$\boxed{3}$ Primary EQ

$A = (x+2)(y+2)$ find y

$\boxed{4}$ Secondary EQ $(A = lw)$

$xy = 30$

$y = \frac{30}{x}$

$A = (x+2)\left(\frac{30}{x} + 2\right)$

$= 30 + 2x + \frac{60}{x} + 4$

$= \frac{60}{x} + 2x + 34 \quad (= 60x^{-1} + 2x + 34)$

use for derivative

$\boxed{5}$ domain:

x must be positive

$1 \le x \le 30$

$\boxed{6}$ $A' = \frac{-60}{x^2} + 2 = \frac{-60 + 2x^2}{x^2}$

$0 = -60 + 2x^2$

$60 = 2x^2$

$30 = x^2$

$\sqrt{30} = x$ critical #

$0 = x^2$

* remember x has to positive so $x = 0$ not in domain

Sign Test

$+ \quad | \quad +$

$\sqrt{30}$

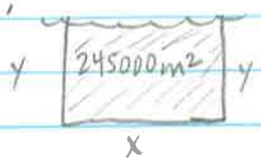
$\therefore \sqrt{30}$ is a min

$\boxed{7}$ $\boxed{4}$ $y = \frac{30}{\sqrt{30}}$ $\boxed{3}$ $(\sqrt{30} + 2) \times \left(\frac{30}{\sqrt{30}} + 2\right)$

The dimensions of the page must be $x = (\sqrt{30} + 2)$ in by $y = \left(\frac{30}{\sqrt{30}} + 2\right)$ in to use the least amount of paper.
7.477   7.477

19.

2



3 primary EQ

$P = x + 2y$

4 Secondary EQ

$xy = 245000$

$y = \frac{245000}{x}$

$P = x + 2\left(\frac{245000}{x}\right) = x + \frac{490000}{x}$

(where P is the length of fence needed)

rewrite for deriv

$x + 490000x^{-1}$

5 domain

$0 < x < 700$

6 $P' = 1 - \frac{490000}{x^2} = \frac{x^2 - 490000}{x^2}$

$0 = x^2 - 490000$ $\quad$ $x^2 = 0$

$x^2 = 490000$ $\quad$ $x = 0$

$x = \pm 700$

only 700 is critical #

Sign Test

$-$ $\quad$ $+$

700

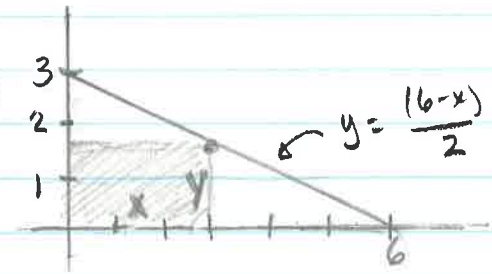
$\therefore$ 700 is a minimum

7 4 $y = \frac{245000}{700} = 350$

The dimensions of the pasture would be x=700m by y=350m to use the least amount of fencing.

22.

$\boxed{2}$



$\boxed{3}$ Primary EQ

$A = xy$

$\boxed{4}$ Secondary EQ

$y = \frac{(6-x)}{2}$

$A = x\left(\frac{6-x}{2}\right) = \frac{6x-x^2}{2}$ ↳ rewrite for deriv $= \frac{1}{2}(6x-x^2)$

(constant multiplier)

$\boxed{5}$ domain

$x > 0$

$\boxed{6}$ $A' = \frac{1}{2}(6-2x) = 3-x$

$0 = 3-x$

$x = 3$ Critical #

Sign Test

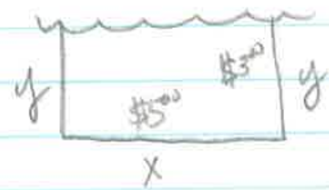
+ | −

3

$\boxed{7}$ $y = \frac{6-3}{2} = \frac{3}{2}$

The dimensions of the rectangle must be $x = 3$ units by $y = 3/2$ units to produce the maximum area.

Australian Rancher Problem



primary EQ

$A = xy$

find y

Secondary EQ

$5x + 6y = 900$

$6y = 900 - 5x$

$y = \frac{900 - 5x}{6}$

$A = x\left(\frac{900 - 5x}{6}\right) = \frac{900x - 5x^2}{6} \xrightarrow{\text{rewrite for deriv}} = \frac{1}{6}(900x - 5x^2)$

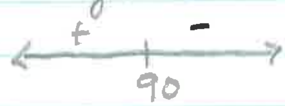
$A' = \frac{1}{6}(900 - 10x) = \frac{300}{2} - \frac{5}{3}x$

$0 = \frac{300}{2} - \frac{5}{3}x$

$\frac{-300}{2} = \frac{-5}{3}x$

$90 = x$

Sign Test



$y = \frac{900 - 5(90)}{6} = \frac{450}{6} = 75$

To produce the maximum area, the dimensions will be x=90ft by y=75ft.