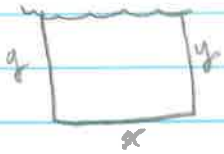
Solutions to worksheet Optimization Problems

1.



$P = 2400$ ft

primary EQ

$A = xy$

Secondary EQ

$x + 2y = 2400$

$2y = 2400 - x$

$y = \frac{2400 - x}{2}$

$$A = x\left(\frac{2400 - x}{2}\right) = \frac{2400x - x^2}{2} \xrightarrow{} \text{rewrite for deriv} \quad \frac{1}{2}(2400x - x^2)$$

$$A' = \frac{1}{2}(2400 - 2x) = 1200 - x$$

$1200 - x = 0$

$1200 = x$

Sign Test

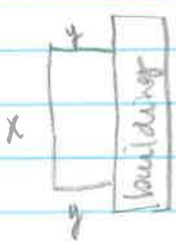
$+ \quad | \quad -$

1200

$$y = \frac{2400 - 1200}{2} = 600$$

The dimensions that would produce the largest area are $x = 1200$ ft by $y = 600$ ft

2.



primary EQ

$A = xy$

Secondary EQ

$x + 2y = 500$

$2y = 500 - x$

$y = \frac{500 - x}{2}$

$$A = x\left(\frac{500 - x}{2}\right) = \frac{500x - x^2}{2}$$

$$A' = \frac{1}{2}(500 - 2x) = 250 - x$$

$0 = 250 - x$

$x = 250$

Sign test

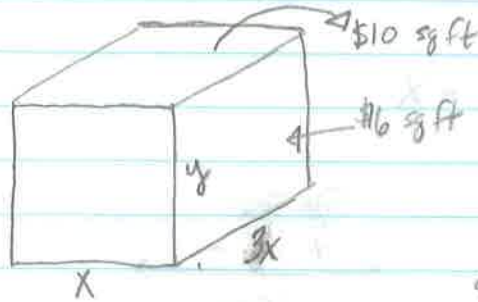
$+ \quad | \quad -$

250

$y = \frac{500-250}{2} = \frac{250}{2} = 125$

The dimensions of the field will be $x = 250$ ft by $y = 125$ ft to produce the maximum area.

3



Primary EQ

$C = 10(2(3x \cdot x)) + 6(2(3x \cdot y)) + 6(2(x \cdot y))$

$= 60x^2 + 36xy + 12xy$

$= 60x^2 + 48xy$

Secondary EQ

$50 = x(3x)(y)$

$\frac{50}{3x^2} = y$

$C = 60x^2 + 48x\left(\frac{50}{3x^2}\right) = \underbrace{60x^2 + \frac{800}{x}}_{\text{rewrite for deriv}} = \frac{60x^3 + 800}{x}$

$60x^2 + 800x^{-1}$

$C' = 120x - \frac{800}{x^2} = \frac{120x^3 - 800}{x^2}$

$0 = 120x^3 - 800 \qquad 0 = x^2$

$800 = 120x^3 \qquad x = 0$

$\frac{80}{12} = x^3$

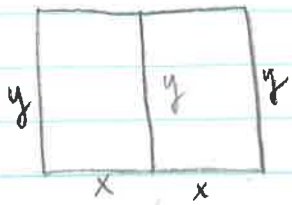
$\sqrt[3]{\frac{80}{12}} = x$

The dimensions of the pasture must be $x = \sqrt[3]{\frac{80}{12}}$ ft by $y = \frac{50}{3\left(\sqrt[3]{\frac{80}{12}}\right)^2}$ ft.

1.882

8.855

4



primary EQ

$A = 2xy$

Secondary EQ

$4x + 3y = 1000$

$3y = 1000 - 4x$

$y = \frac{1000 - 4x}{3}$

$A = 2x\left(\frac{1000-4x}{3}\right) = \frac{2000x - 8x^2}{3}$

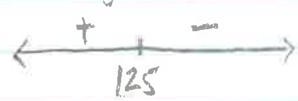
$A' = \frac{1}{3}(2000 - 16x) = \frac{2000 - 16x}{3}$

$0 = 2000 - 16x$

$-2000 = -16x$

$125 = x$

Sign Test



The dimension of the pigpens should be $x = 125$ ft by $y = \frac{500}{3}$ ft. The maximum area of each pigpen
$166\frac{2}{3}$ ft

will be $20{,}833\frac{1}{3}$ ft².